Comparative Analysis of Partial Differential Equation-based and Graph Based Methods for Image Segmentation

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Abstract— In computer vision systems, image segmentation is an important one as far as preprocessing step is concerned. In literature we have several segmentation algorithms, but with constraints due to high processing load and many mammal-tuning parameters. The challenge in image segmentation in images is inhomogeneity levels in intensity. Existing segmentation algorithms depend on region-based and it features similarity of intensities of images in the region of interest. The segmentation results are not accurate due to the presence of non-uniformity in intensities of images. This paper covers segmentation techniques to overcome the intensity inhomogeneity with the help of partial differential equation based methods. This paper critically reviews some of these techniques. It also addresses the quantitative evaluation of segmentation results.

Keywords— Fast Marching, Level set, Parametric, Image Segmentation.

I. INTRODUCTION

In computer vision, the process of partitioning a digital image into multiple segments is called as image segmentation. The goal of image segmentation is to simplify the representation of an image into something that is more meaningful and easier to analyze. Image segmentation methods are used to locate objects and boundaries in images. The process in image segmentation is to label each pixel in an image such that pixels with the same label contain certain observable characteristics. Image Segmentation techniques can be defined as the methods that are used for representing and extracting the information from an image. The accuracy of image segmentation method is determined by the success or failure of an automated analysis procedure. The final result of image segmentation methods is a set of segments that collectively represent the whole image. In a region of an image, each of the pixels is similar with respect to some characteristics or property such as intensity, texture, color. When these methods are applied to a stack of images, (medical imaging), the extracted contours after segmentation can be used to create 3D models. Image segmentation has many applications. The main applications are machine vision, content-based image retrieval, Object detection, Pedestrian detection, Medical Imaging, Face detection, Detection of Brake light, Location of objects in satellite images, Hand written character recognition, Traffic control systems, Iris recognition. The main applications of medical imaging are identifying tumors and other pathologies, measuring the tissue volumes, study of anatomical structure. Segmentation methods aim at dividing an image into regions with different properties as a means for separating foreground and background images. Many recent algorithms (e.g. [8, 16, 17]) have been proposed to use and combine multiple cues, as objects may be separable by different cues (intensity, color, texture, or boundary continuity). In such algorithms, each cue is handled by a separate module. The role of the module is to assess the similarity of nearby pixels or regions according to that cue, by integrating these similarities, segmentation decision is obtained. Though the design of these modules along with the use of appropriate optimization methods led to notable successes, but still there is a challenge in the segmenting objects in a variety of natural images.

Methods based on Front propagation with implicit representation of the evolving front proved have led to convincing results for segmentation purposes [2, 13, 14, 6, and 15]. But considerable processing time is required to solve the underlying partial differential equation especially on 3D images. A wide range of problems in computer vision could make good use of segmented images, were such segmentations reliably and efficiently computable. For instance vision problems of middle level such as motion and stereo estimation require appropriate region of support for correspondence operations. Non-uniform regions of support can be identified using segmentation techniques. High level problems such as image indexing, recognition can make use of segmentation results in matching, to address the problems such as recognition by parts, figure-ground separation.

II. PARTIAL DIFFERENTIAL EQUATION METHODS

A. Parametric Methods

Lagrange methods are based on parameterizing the contour according to sampling strategy and by evolving each element according to image terms and internal terms. Such methods are fast and efficient, however the original parametric formulation [1] is criticized for its limitations due to the selection of sampling technique, the geometric
properties of the curve, changes in topology. Efficient discretized methods were developed to address these limitations while maintaining high efficiency. In both cases, minimization of energy is generally conducted using a steepest descent, whereby using finite differences derivatives are computed.

The work on parametric active contour [1] defined the active contour as an energy minimizing model, \( v(s) = \{x(s), y(s), s \in (0,1)\} \), with normalized arc length \( s \). The aim is to evolve the deformable model to minimize the energy functional

\[
E = \int_0^1 \left( \alpha(s) v_1^2(s) + \beta(s) v_2^2(s) - \gamma(s) E_{ext}(v(s)) \right) ds
\]

where \( v_1 \) and \( v_2 \) are the first and second derivatives of with respect to arc length \( s \), and the parameters \( \alpha(s) \), \( \beta(s) \), and \( \gamma(s) \) are the penalties on slope, curvature and the external force, respectively.

Kernel-based active contour approach to object boundary detection was introduced by [3]. In this an adaptive kernel derived from image characteristics was introduced into the parametric active contour approach, KPAC is able to handle noise, complex curvature boundaries, background clutter. The convergence speed was also improved in this model. KPAC was able to identify the interested boarders, while GVFS and VFC both exhibit errors in the identified boundaries.

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\[\text{Fig.1. Segmentation of the Road and Marmor sequences. (a) Motion map of the Road sequence. (b) Segmentation at convergence of Road sequence.}\]

B. Parametric Distributional Clustering for Image Segmentation

Image Segmentation using unsupervised methods is one of the important issues in Computer Vision. From the exploratory data analysis point of view, segmentation is formulated as a clustering problem in which pixels are grouped together based on local features. Parametrical distributional clustering (PDC) was presented by [4] as a novel approach to image segmentation. In this local distributions of image features provide a robust description of the local image features in contrast to noise sensitive point measurements. The authors have formulated the segmentation methods as a generative model in the maximum likelihood framework. Moreover, there exists a connection to the information theoretic concept of the Information Bottleneck (Tishby et al. [5]), which emphasizes the compromise between efficient coding of an image and preservation of characteristic information in the measured feature distributions. Deterministic annealing techniques are used to solve the optimization problem of good grouping.

C. Level Set Methods

The level set method was proposed by by Osher and Sethian [6] to track moving interfaces in 1988 and has been widely used in various imaging domains. It can be used to address the problem of curve or surface propagation in an implicit manner. In this the evolving contour is represented using a signed function, where its zero level corresponds to the actual contour. It can be observed that, according to the motion equation of the contour, one can obtain a similar flow for the implicit surface that when applied to the zero-level will reflect the propagation of the contour. The level set method has many advantages: it is parameter free, implicit, the geometric properties of the evolving structure can be estimated. They can be used to define an optimization framework which was introduced by Zhao, Merriman and Osher in 1996. Therefore, it can be concluded that it is a sophisticated framework to address various applications of biological image analysis and computer vision. Research into different level set data structures has led to effective implementations of this technique. In level set method the main idea is to represent the contour as a zero level set of a higher dimensional function, called a level set function, and the motion of the contour is formulated as the evolution of the level set function.

DRLSE [7] method has the capability of maintaining the regularity of the level set function, particularly it is a desirable signed distance property in a vicinity of the zero level set, which promises accurate computation and stable level set evolution. DRLSE can be implemented by a simple and efficient numerical method than conventional level set methods. DRLSE allows flexible and efficient initialization.
than generating a signed distance function as the initial level set function. It has been observed that, DRLSE model allows the use of large time steps to reduce iteration numbers and processing time, while maintaining numerical accuracy in full domain and narrowband implementations, due to the intrinsic distance regularization embedded in the level set evolution. Further it is observed that, because of its efficiency and accuracy, regularized level set evolution has been widely used in applications in the area of image segmentation.

Fig. 2. Comparison of the DRLSE model and the GAC model on three synthetic images in (a), (b), and (c).

III. GRAPH BASED METHODS

Graph based methods can effectively be used in computer vision problems especially in the area of image segmentation. In these graph based methods, the image is represented as a weighted, undirected graph. In these methods a pixel or a group of pixels are represented with nodes and edge weights represent the dissimilarity between the adjacent pixels. The graph is then partitioned into clusters. Each partition of the graph resulted from these techniques are considered as an object segment in the image. Some algorithms in this category are minimum spanning tree-based segmentation [12], random walker [9], normalized cuts [8], minimum cut [10], isometric partitioning [11].

A. Normalized Cut

A Novel method for image segmentation based on graphical model was proposed by [8]. In this, a graph \( G \), can be partitioned into two disjoint sets \( P, Q \), such that \( PUQ = G \), \( P\cap Q = \emptyset \), by deleting the edges connecting the two parts. The degree of dissimilarity between these two disjoint sets can be formulated as the total weight of edges that are deleted.

\[
\text{Cut} (P, Q) = \sum_{u \in P, v \in Q} Z(P, Q)
\]

The bi-partitioning of a graph is the one which minimizes the cut.

B. Random Walker

The random walker algorithm is an algorithm for image segmentation. The image is represented as a graph, in which each pixel is represented with a node which is connected to neighboring pixels by edges, and weights which represent the similarity between the pixels are assigned to edges. In this [9] a user manually labels a small number of pixels with seeds as "object" and "background". In this the unlabeled pixels are assumed to release a random walker, and the probability is calculated that each pixel's random walker first arrives at a seed with each label, i.e., if a user places \( L \) seeds, each with a different label, then it is necessary to calculate, for each pixel, the probability that a random walker leaving the pixel will first arrive at each seed. This calculation can be determined by solving a system of linear equations. After the probabilities for each pixel are computed, the pixel is assigned to the label for which it is most likely to send a random walker.

C. Isometric Partitioning

A new algorithm for graph partitioning that attempts to find sets with a low isoperimetric ratio was described by [11]. The algorithm was compared with Normalized cuts to demonstrate that it is faster and stable and requires less pre-processing.
IV. CONCLUSION

In this paper we surveyed different approaches for image segmentation using parametric based methods and graph based methods. Graph based techniques are very useful in biological space variant systems. Random walker method does not suffer from discretization errors.

REFERENCES