THREE DIMENSIONAL LANGUAGE AND ITS GRAMMAR

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Abstract

We describe in this paper a practical problem of covering a rectangular room with many smaller cubes. A context free grammar with parameters are constructed here, Volume of the rectangular room and the sum of volume of cubes used are equal. The source of cubes are considered as two cases, one having height equal and other is of different height. The language contains all the models that can be generated using only Guillotine restrictions. Every model starts with cuts of same volume as the target shape and works out the way of combining the pieces for optimal covering of the room.

1 Keywords

Volume of rectangular room, Guillotine restriction, Height of the room, Tree Diagram, Grammar.

2 Introduction

The problem of cutting and covering arises in various production process with application varying from the home textile to the glass, steel, wood and paper industries, where rectangular figures are cut from large rectangular sheets of materials. Paper(1) describes a context free grammar, with parameters, that generate models for covering a rectangular area with many smaller rectangular pieces. The generated language contains all the models that can be generated using only Guillotine restrictions.

In the light of the paper we are trying to discuss the following problem related to our daily life. The problem is to fill a rectangular room with small rectangular boxes. We are given a rectangular box with volume equal to the volume of the rectangular room. We want to cover the room using the rectangular box with minimum loss. We got a model by cutting the room from one boarder to other of the rectangular room. The three cases relating to the measures that is case 1 : when \( h_1 = h_2 \)

\( \text{case 2: when } h_1 < h_2 \text{ case:3 when } h_1 > h_2 \)

where \( h_1 \) and \( h_2 \) are the height of the rectangular room and the box which is used to cover it respectively. In this paper we constructed a context free grammar with parameter that can generate the covering model. We also generate an algorithm for it.

3 Description of the problem

We can consider a rectangular room \( R \) of volume \( V \) (Similar to rectangular box) which is to be filled by smaller rectangular boxes having volume \( V_i \) for \( i=1,2,\ldots,n \). The covering model is such that \( \sum V_i = V \) here we cut the boxes with guillotine restriction.
That is a cut is possible from one border to the other existing border. For example we have to fill a room of dimension 2 meters, 6 meters and 10 meters of length breadth and height respectively with another box of length 3m, breadth 6m and height 10m. We can cover the room with one box by losing the rest of length 1m breadth 6m and height 10m or by cutting into smaller boxes and reducing the loss. To do this without loss we should know how a room of dimension $l_1, b_1, h_1$ can be covered by cutting a box with dimension $l_2, b_2, h_2$ with a condition that $l_1 * b_1 * h_1 = l_2 * b_2 * h_2$. Here we consider the three possible cases. That is when $h_1 = h_2, h_1 > h_2$ and $h_2 > h_1$. In all these three cases we cover the rectangular room by cutting the rectangular boxes and laying them in two directions, say horizontal direction and vertical direction.

**Example: 1.** Let $l_1 = 6, b_1 = 10, h_1 = 4, l_2 = 4, b_2 = 15$ and $h_2 = 4$. Then $l_1 * b_1 * h_1 = l_2 * b_2 * h_2 = 240$. In the first case (figure 1) we cover in the horizontal direction.

Now $4m * 10m * 4m$ of the room is covered and the remaining $2m * 10m * 4m$ is to be covered with remaining box of dimension $4m * 5m * 4m$. Still we have to cover the room of volume $2m * 5m * 4m$ using a box of volume $2m * 5m * 4m$. Thus we can cover the room by the rectangular box without loss.

## 4 Construction of Grammar

Let $l_1, b_1, h_1$ be the dimensions of the whole rectangular room which is to be covered using a rectangular box of dimension $l_2, b_2, h_2$. In the constructional grammar we consider the following.

1. **Horizontal direction**

a) $l_1 > l_2$

b) $l_2 > l_1$

2. **Vertical direction**

a) $l_1 < b_2$

b) $b_2 < l_1$

**Definition:** 1. Context free grammar $G$ with parameters $(l_1, b_1, h_1, l_2, b_2, h_2)$ is $G(l_1, b_1, h_1, l_2, b_2, h_2) = (N, T, C, S, P)$ where

$$N = \{S(l_1, b_1, h_1, l_2, b_2, h_2) / (l_1, b_1, h_1, l_2, b_2, h_2) \in R\}$$

is the set of variables (non terminals)

$$T = \{(l_1, b_1, h_1, l_2, b_2, d) / (l_1, b_1, h_1, l_2, b_2, h_2) \in R, d \in \{\rightarrow, \uparrow\}\}$$

is the set of terminals

$$C = \{(l_2, b_2, h_2, \partial y) / (l_2, b_2, h_2) \in R, \}$$

$\partial y \in \{>, <, \leq, \geq, =\}$ is the set of conditions $S = S(l_1, b_1, h_1, l_2, b_2, h_2)$ is the initial symbols where $l_1, b_1, h_1, l_2, b_2, h_2$ are the grammars parameter.

$P = N \rightarrow N \times T$ is the set of rules.

The rules are of the form

Case 1 when $h_1 = h_2 = h$

$S(l_1, l_2, h, l_2, b_2, h) \rightarrow (l_1, b_1, h, l_2, b_1, h, \rightarrow)$

$S(l_1 - l_2, b_1, h, l_2, b_2 - b_1, h)$

if $l_2 < l_1 \rightarrow (1)$

$S(l_1, b_1, h, l_2, b_2, h) \rightarrow (l_1, b_1, h, l_1, b_1, h, \rightarrow)$

$S(l_1, b_1 - b_2, h, l_2 - l_1, b_2, h)$

if $l_2 > l_1 \rightarrow (2)$

$S(l_1, b_1, h, l_2, b_2, h) \rightarrow (l_1, b_1, h, l_1, l_2, h, \downarrow)$

$S(l_1, b_1 - b_2, h, l_2 - b_2 - b_1, h)$

if $l_2 < b_1 \rightarrow (3)$

$S(l_1, b_1, h, l_2, b_2, h) \rightarrow (l_1, b_1, h, b_2, b_1, h, \downarrow)$

$S(l_1 - b_2, b_1, h, l_2 - b_1, b_2, h)$

if $l_2 > b_1 \rightarrow (4)$

$S(l_1, b_1, h, l_2, b_2, h) \rightarrow (l_1, b_1, h, l_1, b_1, h, \rightarrow)$

if $l_2 =
if $l_1 < l_1 \to (5)$

$S \left( l_1, b_1, h, l_2, b_2, h \right) \to \left( l_1, b_1, h, b_1, l_1, h, \downarrow \right)$

if $l_2 = b_1 \to (6)$

Case 2 When $h_1 \neq h_2$ and $h_2 > h_1$

$S \left( l_1, b_1, h_1, l_2, b_2, h_2 \right) \to \left( l_1, b_1, h_1, l_1, b_2, h_2 \right) \to \left( l_1, b_1, h_1, l_2, b_2, h_2 \right) \to \left( l_1, b_1, h_1, l_1, b_2, h_2 \right)$

if $l_2 < l_1 \to (1)$

$S \left( l_1, b_1, h_1, l_2, b_2, h_2 \right) \to \left( l_1, b_1, h_1, l_1, b_2, h_2 \downarrow \right)$

if $l_2 > l_1 \to (2)$

$S \left( l_1, b_1, h_1, l_2, b_2, h_2 \right) \to \left( l_1, b_1, h_1, l_1, b_2, h_2 \downarrow \right)$

if $l_2 < b_1 \to (3)$

$S \left( l_1, b_1, h_1, l_2, b_2, h_2 \right) \to \left( l_1, b_1, h_1, b_1, l_2, b_2, h_2 \downarrow \right)$

if $l_2 > b_1 \to (4)$

$S \left( l_1, b_1, h_1, l_2, b_2, h_2 \right) \to \left( l_1, b_1, h_1, l_1, b_2, h_2 \downarrow \right)$

if $l_2 < b_1 \to (5)$

$S \left( l_1, b_1, h_1, l_2, b_2, h_2 \right) \to \left( l_1, b_1, h_1, l_2, b_2, h_2 \downarrow \right)$

if $l_2 > b_1 \to (6)$

$S \left( l_1, b_1, h_1, l_2, b_2, h_2 \right) \to \left( l_1, b_1, h_1, b_1, l_2, b_2, h_2 \downarrow \right)$

if $l_2 < h_1 \to (7)$

$S \left( l_1, b_1, h_1, l_2, b_2, h_2 \right) \to \left( l_1, b_1, h_1, b_1, l_2, b_2, h_2 \downarrow \right)$

if $l_2 < b_1 \to (8)$

Context free grammars with parameters generate a language

Definition 2. The generated languages of the context free grammars is defined by

$L(G \left( l_1, b_1, h_1, l_2, b_2, h_2 \right)) = \{ d/d \in T*, S \left( l_1, b_1, h_1, l_2, b_2, h_2 \right) \Rightarrow d \}$

where $d = d_1, d_2, ..., d_n$

Each $d_i$ represent covering of the rectangular room with another rectangular box.

The following observations can be obtained from the above grammar and its production rule.

1. As set of variables $N$ is finite there are a finite number of covering boxes and the number of generating grammars are also finite.

2. If the surface to be covered is of dimension $l_1, b_1, h_1$ and the material used for covering has got dimensions $l_2, b_2, h_2$ then production rule can be applied as follows. The variable $S \left( l_1, b_1, h_1, l_2, b_2, h_2 \right)$ is replaced by the rule

$S \left( l_1', b_1', h_1', l_2', b_2', h_2' \right)$

where $l_1', b_1', h_1'$ represents the dimension of the box that is used to cover the surface. $l_1', b_1', h_1'$ represents the surface remaining to be covered. $l_2'', b_2'', h_2''$ represents the remaining dimensions of the box which is used for covering. 'd' is the direction on which the surface is being covered.

3. Covering always done from one boarder to the other boarder.

4. The grammar gives a one dimensional representation of a three dimensional box that represents a covering model.

A tree model can be constructed for the context free grammars with parameters.

Definition 3. For a context free grammar with parameters a binary tree
model can be constructed by labelling each of its nodes with a symbol from $T$ and corresponding to each production rule say $S(l - 1, b_1, l_1, l_2, b_2, h_2) \rightarrow (l_1, b_1, l_1', b_2', h_2', d)S(l_1', b_1', l_2'', b_2'', h_2'')$. The left child is labelled with the terminals $S(l_1, b_1, l_1', b_2', h_2', d)$ and the right child is labelled with the variable $S(l_1', b_1', l_2'', b_2'', h_2'')$.

The following example shows the classification for the above mentioned facts.

**Example: 2.** Let $l_1 = 5, b_1 = 8, h_1 = 10$ be the dimensions of the rectangular room and the dimensions of the rectangular box used for covering the room be $l_2 = 2, b_2 = 20, h_2 = 10$.

The production rules are given as follows. Above each $\Rightarrow \text{ symbol}$ we write the production rule number. Here $l_1 = h_2$. So we apply production rule corresponds to case 1.

$S(5, 8, 10, 2, 20, 10) \Rightarrow (5, 8, 10, 2, 8, 10, \rightarrow)S(3, 8, 10, 2, 12, 10)$  
$\Rightarrow (5, 8, 10, 2, 8, 10, \rightarrow)(3, 8, 10, 3, 2, 10, \downarrow)S(3, 6, 10, 2, 9, 10)$  
$\Rightarrow (5, 8, 10, 2, 8, 10, \rightarrow)(3, 8, 10, 3, 2, 10, \downarrow)S(3, 6, 10, 2, 3, 10)$  
$\Rightarrow (3, 6, 10, 2, 6, 10, \rightarrow)S(1, 6, 10, 2, 3, 10)$  
$\Rightarrow (5, 8, 10, 2, 8, 10, \rightarrow)(3, 8, 10, 3, 2, 10, \downarrow)S(3, 6, 10, 2, 3, 10)$  
$\Rightarrow (3, 6, 10, 2, 3, 10, \rightarrow)S(1, 3, 10, 1, 3, 10)$  
$\Rightarrow (5, 8, 10, 2, 8, 10, \rightarrow)S(1, 6, 10, 1, 3, 10)$  
$\Rightarrow (5, 8, 10, 2, 8, 10, \rightarrow)(3, 6, 10, 2, 6, 10, \rightarrow)(1, 6, 10, 1, 3, 10, \rightarrow)S(1, 3, 10, 1, 3, 10)$  
$\Rightarrow (5, 8, 10, 2, 8, 10, \rightarrow)S(1, 6, 10, 1, 3, 10, \rightarrow)S(1, 3, 10, 1, 3, 10, \rightarrow)$.  

The tree derivation is shown below.

![TREE.png](image_url)

The three dimensional picture showing the resultant covering of the room is given in fig 5.

**Example: 3.** Let $l_1 = 5, b_1 = 8, h_1 = 10$ be the dimension of the dimension of the rectangular room and $l_2 = 2, b_2 = 10, h_2 = 20$ be the dimensions of the rectangular box used for covering the room. As described in above example the production rule number is written above each $\Rightarrow \text{ symbol}$.

$\Rightarrow (5, 8, 10, 2, 10, 20) \Rightarrow (5, 8, 10, 2, 10, 20, \rightarrow)S(3, 8, 10, 2, 2, 10) \Rightarrow (5, 8, 10, 2, 10, 20, \rightarrow)(3, 8, 10, 3, 2, 10, \downarrow)S(3, 6, 10, 2, 7, 10)$  
$\Rightarrow (5, 8, 10, 2, 10, 20, \rightarrow)(3, 8, 10, 3, 2, 10, \downarrow)S(1, 6, 10, 2, 1, 10)$  
$\Rightarrow (5, 8, 10, 2, 10, 20, \rightarrow)S(1, 6, 10, 1, 2, 10)$  
$\Rightarrow (5, 8, 10, 2, 10, 20, \rightarrow)(3, 8, 10, 3, 2, 10, \downarrow)S(3, 6, 10, 2, 6, 10, \rightarrow)(1, 6, 10, 1, 2, 10, \downarrow)$  
$\Rightarrow (5, 8, 10, 2, 10, 20, \rightarrow)S(3, 8, 10, 3, 2, 10, \downarrow)$  
$\Rightarrow (5, 8, 10, 2, 10, 20, \rightarrow)(3, 8, 10, 3, 2, 10, \downarrow)$  
$\Rightarrow (5, 8, 10, 2, 10, 20, \rightarrow)(3, 8, 10, 3, 2, 10, \downarrow)$  

The derivation tree is shown below.

![The picture showing the resultant covering of the room is given in the figure](image_url)
Example: 4. Let \( l_1 = 2, b_1 = 10 \) and \( h_1 = 20 \) be the dimension of the rectangular room and let \( l_2 = 5, b_2 = 8 \) and \( h_2 = 10 \) be the dimension of the rectangular box used for covering the room. Production rules are given below.

\[
S(2, 10, 20, 5, 8, 10) \implies (2, 10, 20, 2, 8, 10, \rightarrow) \\
S(2, 20, 3, 8, 10) \implies (2, 10, 20, 2, 8, 10, \leftarrow) \\
(2, 10, 3, 2, 10, \ll) S(2, 7, 20, 3, 6, 10) \\
\implies (2, 10, 20, 2, 8, 10, \rightarrow) (2, 10, 20, 3, 2, 10, \ll) \\
(2, 7, 20, 2, 6, 10, \ll) S(2, 1, 20, 1, 6, 10) \\
\implies (2, 10, 20, 2, 8, 10, \rightarrow) (2, 10, 20, 3, 2, 10, \ll) \\
(2, 7, 20, 2, 6, 10, \ll) (2, 1, 20, 1, 2, 10, \rightarrow) \\
S(2, 2, 10, 1, 4, 10) \implies (2, 10, 20, 2, 8, 10, \rightarrow) \\
(2, 10, 20, 3, 2, 10, \ll) (2, 7, 20, 2, 6, 10, \ll) \\
(2, 1, 20, 1, 2, 10, \rightarrow) (2, 2, 10, 1, 2, 10, \rightarrow) \\
S(1, 2, 10, 1, 2, 10) \\
\implies (2, 10, 20, 2, 8, 10, \rightarrow) \\
(2, 10, 20, 3, 2, 10, \ll) (2, 7, 20, 2, 6, 10, \ll) \\
(2, 1, 20, 1, 2, 10, \rightarrow) (2, 2, 10, 1, 2, 10, \rightarrow) \\
(1, 2, 10, 1, 2, 10, \rightarrow)
\]

The derivation tree is shown below.

The three dimensional picture showing the resultant covering of the room is given in figure below.

5 Algorithm

Input: The rectangular room \( R_1 \) or the rectangular box \( R_2 \) required for the problem

Output: Small rectangular box representation of the cutting pattern whose total volume is equivalent to the volume of the rectangular room.

Procedure \((R_1, R_2)\): The algorithm constructs a tree diagram for the cutting pattern. Starting from the root to the leaves. Each vertex of the tree gives the information whether it is possible to make guillotine movement in horizontal or vertical direction.

Procedure: Cover \((R_1, b_1, l_1, l_2, b_2, h_2, d)\)

For horizontal direction

begin

If \( l_1 < l_2 \)

\((l_1, b_1, l_1, l_2, b_2, h_2, d) \rightarrow (l_1, b_1 - b_2, h_1, l_2 - l_1, b_2, h_2 - h_1)\)

end

else if \( l_2 < l_1 \)

begin

\((l_1, b_1, l_1, l_2, b_2, h_2, d) \rightarrow (l_1 - l_2, b_1, h_1, l_2, b_2 - b_1, h_2 - h_1)\)

end

For vertical direction

begin

If \( l_2 < b_1 \)

\((l_1, b_1, l_1, l_2, b_2, h_2, d) \rightarrow 3(l_1, b_1 - b_2, h_1, l_2, b_2 - b_1, h_2 - h_1)\)

end

else if \( l_2 > b_1 \)

begin

\((l_1, b_1, l_1, l_2, b_2, h_2, d) \rightarrow (l_1 - l_2, b_1, h_1, l_2 - l_1, b_2, h_2 - h_1)\)

end

If \( b_1 < b_2 \) and \( l_1 = l_2 \)

begin

\((l_1, b_1, l_1, l_2, b_2, h_2, d) \rightarrow \)
(l₁, b₂ − b₁, h₁, l₂, b₂, h₂ − h₁)
end
else
If b₂ < b₁ and l₁ = l₂
begin
(l₁, b₁, h₁, l₂, b₂, h₂, d) →
(l₁, b₁, h₁, l₁, b₂ − b₁, h₂ − h₁)
end
else
if b₁ < b₂ and b₁ = l₂
begin
(l₁, b₁, h₁, l₂, b₂, h₂, d) →
(l₁ − b₂, b₁, h₁, l₂, b₂, h₂ − h₁)
end
else
if b₂ < b₁ and b₁ = l₂
(l₁, b₁, h₁, l₂, b₂, h₂, d) →
(l₁ − b₂, b₂, h₁, l₂, b₂, h₂ − h₁)
end
return
end (Cover)

6 Conclusion

The two dimensional and the three dimensional packing and covering problem holds importance to many fields. The two dimensional covering problem is used in the paper industry or glass industry or in the field of pattern recognition. Architecture engineering and design are certain areas where three dimensional packing and covering problems could apply. One of the important application is in the wood industry.

In this paper we constructed a three dimensional language for the covering models using guillotine restriction.

References


