Design of Robust Controller for Uncertain System

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Abstract- Spherical tank systems are characterized by high uncertainty which makes hard to maintain its good stability margins and performance for closed loop system [7]. In conventional control, if the system parameter changes we cannot guarantee about the system performance hence it is important to design robust control for uncertain plant. In this paper the Quantitative Feedback Theory (QFT) has proved its dominance especially in the case of significant parametric uncertainty. The feature of Quantitative Feedback Theory is that it can take attention of huge parametric uncertainty along with phase information. For the purpose of QFT, the feedback system is usually defined by the two-degrees-of freedom arrangement. Here in the Spherical tank system the height can be controlled by manipulating the feed In-flow rate of the system.

Keywords-Quantitative Feedback Theory, Spherical Tank System, Loop Shaping, Robust Control.

I. Introduction

Robust control is the branch of control engineering that deals with analysis and synthesis of uncertain feedback systems. A number of synthesis methodologies in robust control have been developed over the past few decades, for example those based on H∞, structured singular value and quantitative feedback theory. There are various nonlinear systems in process industries. These kind of nonlinear systems exhibit various challenging control problems due to their nonlinear dynamic behaviour time varying and uncertain parameters, constraints on manipulated variable, interaction between controlled variables and manipulated, frequent disturbances and unmeasured, dead time on input and measurements. The spherical tank exhibits non-linearity because of its change in area. Designing a controller for a nonlinear system is hard and difficult to implement. The main task of the controller is to maintain the process at the preferred operating conditions and to achieve the best performance when facing different types of disturbances. Spherical tanks find wide application in chemical industries. Control of a level in a spherical tank is essential, because the change in area gives rise to nonlinearity. This nonlinearity in the system makes the control procedure complex. It is not easy to bring a non-linear process to steady state. We have to carefully choose the initial settings & see to that there is no much fluctuations in the power supply which adds to the problem of non-linearity. So here we describe the process of designing the controller for the spherical tank using Quantitative Feedback Theory. Quantitative Feedback Theory (QFT) is well-established body of robust feedback synthesis techniques, capable of dealing with large classes of linear and non-linear plants. QFT has been under continuous development since 1959 and currently handles several problem classes such as SISO and MIMO, linear and nonlinear, lumped and distributed, time variant and time invariant system. In QFT, the designer quantitatively formulates the specification on the uncertainty and output tolerances, over the range of design frequencies. The objective of the design is then to guarantee that at every design frequency, the output is member of the tolerable set when the uncertainty belongs to its corresponding set.

II. System specification

The laboratory set up of the system consists of a spherical tank, water reservoir, pump, rotameter, pressure transmitter, electro pneumatic converter pneumatic control valve, interfacing module and Personal Computer (PC). The pressure transmitter output is interfaced with the computer in the RS-232 port of the PC. The MATLAB programs written in script code are linked via the interface module. Thus the setup is interfaced with PC through Data Acquisition card with the help of MATLAB software. The controller is interfaced with the setup and the final control action will be implemented. Fig. 1 shows the real time experimental setup of a spherical tank. The pneumatic control valve is air to adjust the flow of the water pumped to the spherical tank from water tank. The level of the water in the tank is measured by means of the pressure transmitter and it is transmitted in (4-20) mA form to the interfacing module and later to the PC. After computing the control algorithm in the PC control signal is
transmitted to the current to pressure converter in the form of current signal (4-20) mA, which transmit the air signal to the pneumatic control valve. The pneumatic control valve is activated by this signal to produce the required flow of water in and out of the tank. There is irregular flow of water in and out of the tank. Due to the nonlinear physical structure of the spherical tank, the level of water does not maintain itself in a particular level. So the controller is implemented to take necessary control action so that the liquid level gets settled at a particular level within a short period of time.

### IV. TABLE I

**Spherical tank specification**

<table>
<thead>
<tr>
<th>SECTIONS</th>
<th>SPECIFICATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical tank</td>
<td>Diameter &amp; height : 50 cms</td>
</tr>
<tr>
<td></td>
<td>Material : Stainless steel</td>
</tr>
<tr>
<td>Level Transmitter</td>
<td>Pressure transmitter</td>
</tr>
<tr>
<td>Pump</td>
<td>Centrifugal pump</td>
</tr>
<tr>
<td>Control valve</td>
<td>Pneumatic valve</td>
</tr>
<tr>
<td>Rotameter</td>
<td>(0-1000) LHP</td>
</tr>
<tr>
<td>I/V, V/I converter</td>
<td>(4-20) mA to (0-5)V &amp; vice versa</td>
</tr>
<tr>
<td>I/P converter</td>
<td>(4-20) mA to (5-10) V</td>
</tr>
<tr>
<td>Interface</td>
<td>Interfacing module (MATLAB compatible) Connecting system to PC</td>
</tr>
</tbody>
</table>

**Fig.1 Real time experimental setup of a spherical tank**

### V. Mathematical modeling

Any system, if has to be analysed, must be mathematically modelled, i.e. the mathematical equations describing the system must be derived so as to study the system, its features, predicting the dynamic behaviour & for many other purposes. The mathematical modeling of our system is as follows. Consider a spherical tank of radius R. The water flows in at a rate Fin & flows out at a rate Fout.

### VI. Derivation for mathematical model

Any system which has to be analyzed must be mathematically modeled.

Volume of a sphere is given by,

$$V = \frac{4}{3} \pi R^3$$

The first order differential equation of the system is given by,

$$Fin - Fout = \pi R^2 \left[ 1 - \frac{(R - h)^2}{R^2} \right] dh \ dt$$

The system transfer function approximates to a first order plus dead time (FOPDT) equation.

i.e., Transfer function $G(s) = \frac{K e^{-L s}}{T s + 1}$

Where, $K$ = process gain

$T$ = time constant

$L$ = dead time

Using the real time open loop the transfer function for varies height are found shown in Table 2.

### VII. TABLE I

**Transfer function calculation**

<table>
<thead>
<tr>
<th>Height (cm)</th>
<th>Gain</th>
<th>Time constant</th>
<th>Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>2.02</td>
<td>194.01</td>
<td>17.01</td>
</tr>
<tr>
<td>20</td>
<td>3.42</td>
<td>486</td>
<td>17.75</td>
</tr>
<tr>
<td>30</td>
<td>4.5</td>
<td>465</td>
<td>17.5</td>
</tr>
<tr>
<td>40</td>
<td>4.52</td>
<td>547.5</td>
<td>17.9</td>
</tr>
</tbody>
</table>

### VIII. Quantitative feedback theory

**A. Plant description**

The transfer functions are found using the real time open loop test for the spherical tank.

Transfer function $= \frac{K e^{L s}}{T s + 1}$

It has the parametric uncertainty of

$K$ which varies from [2.02……4.52]
L which varies from 
\[ [17.01 \ldots \ldots 17.9] \]
T which varies from 
\[ [194.01 \ldots \ldots 547.5] \]

From the above parametric uncertainty the transfer function is 
\[
\frac{2.02, 4.52}{194.01, 547.5} s + 1
\]
The above uncertainty data should be entered in plant definition window in QFT tool box.

**B. Loop shaping**

Loop shaping is the trail and method of designing the controller for the uncertain system. Care should be taken that the nominal loop lies near the bounds, more specifically above the solid line bounds and below the dashed line bounds at each frequency of interest. Fig.2 shows the loop shaping window with adjusted designed controller.

From the Fig.3 we find the solution for the spherical tank controller is

\[
C(s) = \frac{1.6s + 0.08}{7.407s^2 + 1.5}
\]

Simulink model for the designed controller and the uncertain parameter for upper and lower bounds are shown in Fig.3. The blue line shows the stability of lower boundary and the black line shows the stability of the upper boundary of the uncertain system to be controlled. Here the spherical tank system is controlled for the height 40 cm.

**IX. Conclusion**

In conventional control, if the system parameter changes we cannot guarantee about the system performance. Hence it is essential to design robust control for uncertain plant. The modern control systems like H∞ and μ-synthesis we can’t handle large uncertainty and are applicable to single input single output (SISO) LTI systems. The drawbacks of conventional and modern control theory are excluded by classical control theory based method which can handle large parameter uncertainty and works in frequency domain called as QFT. We have designed controller using QFT for Spherical Tank system with a specified tracking requirements.

**REFERENCE**


